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Membrane vibration experiments: An historical review and recent results

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Abstract

Although dozens of theoretical studies have been published on membrane vibrations, very little experimental work exists in the literature. We provide here a concise review of the experimental membrane vibration literature. A membrane by definition has insignificant bending stiffness. From a vibration point of view, this in effect decouples domains of the membrane from one another in transverse displacement. In this article we show both experimentally and theoretically that this unique character allows for a local response to local excitation. Practical applications of these results may find value in the control-structure-interaction community.

1. Introduction

The study of vibrating membranes goes back at least three centuries. Motivations for such studies were the solution of practical problems; a rich example is the investigation of acoustics of musical instruments such as drums and bells. In modern times, membranes have provided a canonical formalism for mathematical analysis, due to their vanishing thinness and resultant absence of any bending rigidity. Currently, numerous practical applications exist for membrane structures, and are in fact growing in importance. Examples are architectural and civil structures, diaphragms in switches and transducers, biomedical prosthesis such as artificial arteries and organs, and space-based applications such as radio antennas and optical reflectors.

Dozens of theoretical membrane vibrations studies exist in the literature. These cover linear and nonlinear models, various shapes and boundary configurations, and numerous analysis methods including closed-form, asymptotic expansions, and numerical methods (FEM, BEM, etc.). Such studies are ongoing and of current interest.

However, probably less than two dozen fundamental experimental membrane vibration studies can be found in the same literature. Even the simplest classical cases have not been thoroughly investigated. This is due at least in part to the extreme flexibility and lightness of membranes and the noncontact measurement

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methods thus implied. What data that does exist is significantly limited, inaccessible, and insufficient for use in validating theoretical results.

Moreover, pressing questions in modern applications need to be addressed. Interesting and important problems currently exist such as: how do effects like wrinkling, thermal loading, and manufacturing variables affect the vibrational response of membrane/inflatable ''gossamer'' space structures? Can adaptive control methods use vibration information for local and global state estimates in such structures [1]?

In this paper, we first provide a brief review of membrane structures and their applications. A concise historical review of experiments in membrane vibrations follows. Then we present some new experimental results for a vibrating circular membrane, measured using a noncontact scanning laser vibrometer. Finally, we show how the linear vibration theory supports the novel observations.

2. Membrane structures

Structures formed from thin-walled material can be found in a diverse array of applications, from biological organisms, to architectural structures and aerospace craft [2]. Depending on the degree of bending resistance inherent in such structures, as well as the degree of participation of tangential and bending reactions to loads, for certain analytical purposes they may be modeled successfully as membranes, whose unique feature is the absence of any resistance to bending in the form of internal moments.

An interesting example of advanced applications for membrane structures is their use in space. The seminal application of membrane structures for space-based communications occurred over four decades ago with the Echo series of satellites. Recent years have seen a resurgence of interest in membrane structures for extraterrestrial use, due to their potential for reduced launch mass and stowed volume. Applications for such structures range from planar configurations in solar sails, concentrators, and shields, to inflatable lenticulars for radar, radio, and optical uses (see Ref. [3]).

Three key factors are paramount for the success and user acceptance of this developing technology: deployment, longevity, and performance. Performance hinges critically on the precision of the membrane surface. The amount of precision required is highly mission dependent, and may entail one or more of the following measures: surface smoothness, deviation from desired surface profile, and slope error. A range of precision requirements exists. At one end are the solar sails and planar concentrators that require maximum exposed surface area and/or flatness. At the other extreme, membrane optical reflectors may require ratios of aperture diameter to figure error (rms) around $10^6 - 10^7$ or more.

The vibration analysis for such structures is important for at least two critical reasons. One is the usual need to design for vibration reduction or isolation in precision devices; implications of this work will seed future answers to important questions such as modal response variations due to wrinkling, thermal distortion, and manufacturing effects such as local stiffening (e.g., seams). The other reason relates to active shape control of the membrane surface by use of dynamic response for global and local state estimates; use would be made of an on-board laser vibrometer, not unlike that used in this work.

3. An historical review of membrane vibration experiments

Although the theoretical study of vibrating membranes dates from the 1700s, there have been only a handful of experimental studies reported in the literature. This is principally a result of the experimental difficulties that both the extreme low mass and thinness of membrane structures present. Standard methods of experimental modal analysis, for example, the attaching of accelerometers and shaker stingers directly to the vibrating structure, cannot be used with membranes, since such techniques lead to nonnegligible added mass and stiffness that irrevocably pollutes the results. Hence it is not surprising that even relatively simple phenomena, such as the global/local behavior of membrane vibrations discussed later, have not been reported experimentally. Only recently, with the advent of modern optical measurement technology, particularly the scanning laser vibrometer, have such observations been possible. The motivation for what follows in this section is three-fold: (1) to put in one place a reasonably complete and concise presentation of the experimental membrane vibration record; (2) to celebrate the exceedingly clever experimental methods developed over time to address the membrane vibration challenge for noninvasiveness; and (3) to put into context our own observations of an un-reported membrane vibration phenomenon.

Although Ernst Chladni's (1756–1827) work involved vibrating plates, the experimental methods he developed (using sand, filings, and powder to expose the mode shapes of the vibrating plates) were essential for early experiments in membrane vibrations. Moreover, his observation that shavings from the horsehair bow used to excite the plates collected not only with the sand along nodal lines but at anti-nodes as well, motivated in part the earliest reported membrane vibration experiments.

Felix Savart (1791–1841), in a paper to the Royal Academy of Sciences in 1827 [4], discussed Chladni's "subdivision of vibrating sonorous bodies" by experimenting with membranes, among others devices, using Chladni's techniques. It was left, however, to Michael Faraday (1791–1867) to provide the correct explanation of this phenomenon [5]. To this end, Faraday also experimented with membranes: "The effects under consideration are exceedingly well shown and illustrated by membranes. A piece of parchment was stretched and tightly tied whilst moist, over the aperture of a funnel five or six inches in diameter." The membrane was excited by a horsehair filament and a fine lycopodium powder spread over its surface. The distribution of the fine powder to the center of the membrane was correctly explained to be due to air currents resulting from the fluid-vibrating structure interaction.

By the time Bourget performed his membrane experiments (ca. 1860), the linear theory of vibrating membranes was well known (given initially by Euler in 1767). Paper was again used as the membrane, being first wetted and then glued to a wooden frame, with subsequent drying providing the necessary tension. Rayleigh recounts some of Bourget's problem in membrane experiments, including membrane tearing upon drying and change of pitch (tension) with moisture [6,7]. Bourget's membranes were acoustically excited with organ pipes, and again the methods of Chladni were used. He was able to show good qualitative mode shape agreement between theory and experiment, at least for the first few lower modes, but apparently the frequencies did not match so well. Rayleigh [6] discusses several reasons for this observation, including the added mass of surrounding air, and less than perfect boundaries and membrane flexibility. Bernard and Bourget also experimented with square membranes [8].

There appear to be no membrane vibration experiments reported in the literature during the first half of the 20th century. A wonderful experimental paper appeared in 1956 by Bergmann [9], who examined the vibrations of soap films. These are of course not the solid membranes heretofore discussed, but membranes formed from a slurry. Due to the well-known nonuniform thickness of soap films, Bergmann's experimental apparatus provided for rotation of the films to make them uniform. His apparatus used acoustic excitation and a white light illumination system. Interestingly, only symmetric modes are reported in the paper and no mention is made of this fact; presumably, the rotation makes the symmetric modes dominant. In 1964, we find an excellent experimental paper on the nonlinear vibration of membranes [10]. In order to study the effects of large displacement on frequency, an experiment was set up using acoustic excitation and a capacitance displacement sensor to measure the fundamental mode of a 6 μm thick Mylar circular membrane. The fundamental frequency was shown to increase as a function of increasing vibration amplitude.

NASA undertook an experimental investigation of membrane vibrations in 1983 [11]. A three-cornered membrane of order 1 m on a side was suspended vertically in a frame and then excited by a shaker. Modal frequencies and shapes were measured using an eddy current probe mounted on a track. Tests were preformed both in air and in vacuum for various membrane pretensions. Considerable experimental details and data are presented in the report.

In the last dozen years or so, there has been a renaissance of interest in membrane vibrations, motivated in large part by the emergence of "gossamer spacecraft". These ultra-lightweight membrane structures are usually lightly pretensioned (to reduce the boundary mass), and hence their vibratory behavior is especially of interest. Membrane antenna, booms, mirrors, solar sails, and solar shades have been investigated. The annual AIAA Gossamer Spacecraft Forum has been a collection point for presentation of these works, and several experimental efforts have been reported there and in AIAA journals [12–30].

Recently Jenkins and co-workers performed experiments using local excitation on low tensioned membranes [31,32]. They found that, at certain frequencies, the response was itself highly localized. We present below an analysis and discussion of these experimental results.

In summary, several key findings from history may be indicated: (1) novel experimental methods have evolved over time in order to perform membrane vibration experiments with minimal invasiveness, (2) experiments have both confirmed membrane vibration theory and led to new results requiring advances in the theory, and (3) experiments are playing an ever greater role as membranes become increasingly important to structures technology.

4. Experimental investigation of a circular membrane

The experimental setup consisted of a circular membrane of 0.700 m (27.5 in) diameter. The membrane was Mylar polyester with the following properties: thickness (h) = 25.4 μm (0.001 in), Young's modulus (E) = 3.5 GPa (5×10^5 psi), mass density = 1400 kg/m^3 (1.31×10^{-4} lb s^2/in^4), Poisson's ratio (ν) = 0.4, and medium: air.

A 38 mm (1.5 in) diameter loudspeaker was used to acoustically excite the membrane over a localized area at its center. The structure area was thus about 340 times the loading area. Fig. 1 shows a line diagram of the experimental setup.

The clamping fixture consisted of a back plate and clamp ring (see Fig. 1 for details). The back plate included vacuum channels that held the membrane to the plate during mounting and tensioning. Once the membrane was smooth and flat, a clamp plate was mechanically affixed to capture the membrane in place. The membrane was initially assembled with the back plate in a horizontal position. The amount of pre-tension applied was only enough to counteract the membrane sag due to self-weight. Although the exact value of tension was unknown, a reasonable bound can be made. It is easy to show that the maximum stress due to self-weight in a membrane strip held vertically depends only on the mass density and length of the strip and the local gravitational acceleration. In the present case, the maximum stress due to self-weight should not exceed that in a strip of length equal to the membrane diameter, resulting (by Eq. (2) to be introduced later) in a fundamental mode of less than 3 Hz, which is consistent with experimental observation.

A laser vibrometer was used to measure the frequencies and mode shapes. The laser vibrometer (OMETRON VPI 4000) is principally a velocity measuring instrument. It measures the shift in frequency between the incident and reflected laser beams due to the relative motion of the surface (Doppler shift). Under harmonic excitation, the surface velocity is readily related to the surface frequency. The principle can also be applied to random (multi-sine) excitation. In addition, the system uses a set of rotating mirrors to scan points on the surface. Phase relationships among the points are maintained, and thus mode shape generation is possible.

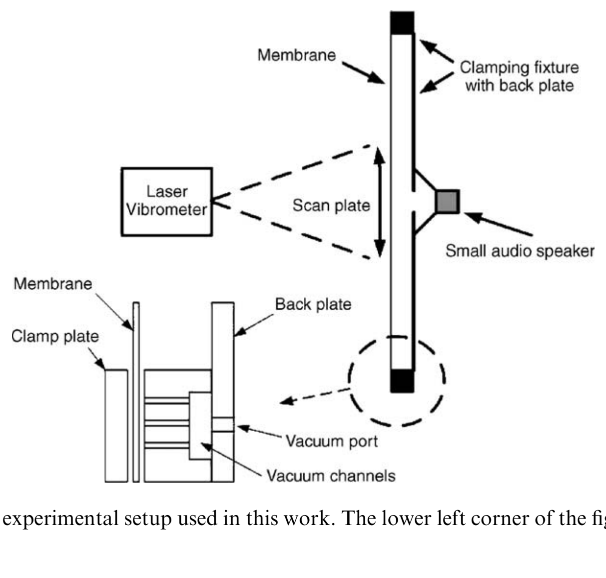


Fig. 1. A schematic diagram of the experimental setup used in this work. The lower left corner of the figure shows details of the membrane clamping fixture and back plate.

In the present case, a random excitation of 0–5000 Hz multi-sine wave signal was first fed to the loud speaker. The laser beam from the vibrometer was placed at different locations on the membrane, and the power spectrum of each of these locations was extracted. The most commonly occurring peaks on the power spectrum were recorded. Two recurring peaks were found at 158 and 1430 Hz. The corresponding power spectrum of the membrane is shown in Fig. 2. Next, a monochromatic excitation (lock-in scan) of the membrane was performed at these two frequencies. A lock-in scan comprises of exciting the membrane at a fixed frequency and scanning its surface with the laser beam. This procedure allows for extraction of the velocity contour and phase information (vibration shape) of the vibrating membrane.

The results of the lock-in scan of the membrane are shown in Figs. 3 and 4. It is evident that at the lower frequency, no discernable localization of vibration response is observed, i.e., the mode shape at 158 Hz is a global mode. However, at the higher frequency, the local nature of the response is striking. These figures

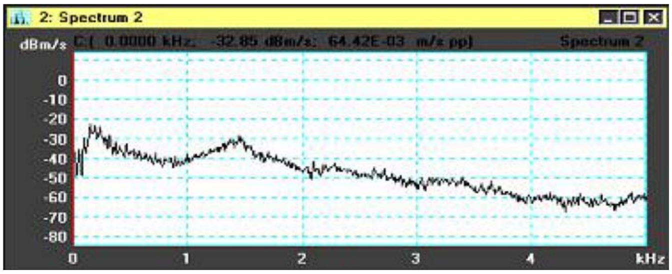
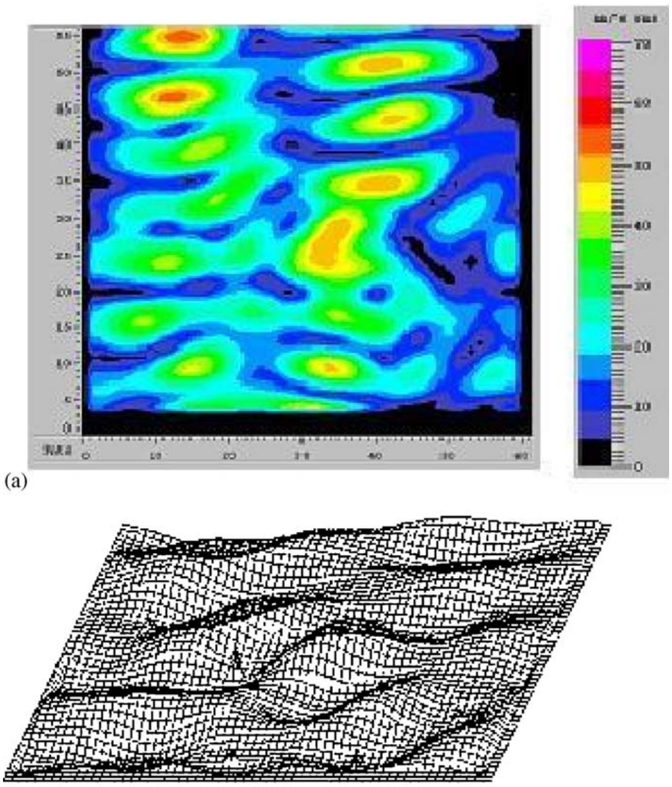


Fig. 2. Power spectrum of the circular membrane. Peak 1: 158 Hz, −35 dB m/s, 50.3×10^{-3} m/s p−p; Peak 2: 1.43 kHz, −32.2 dB m/s, 69.27×10^{-3} m/s p−p.



(a)

(b)

Fig. 3. Membrane vibration shape at frequency = 158 Hz: (a) velocity contour and (b) corresponding vibration shape of the central 300 mm × 300 mm (12 in × 12 in) region; the global nature of the response (to excitation by the 38 mm diameter speaker) is clearly evident.

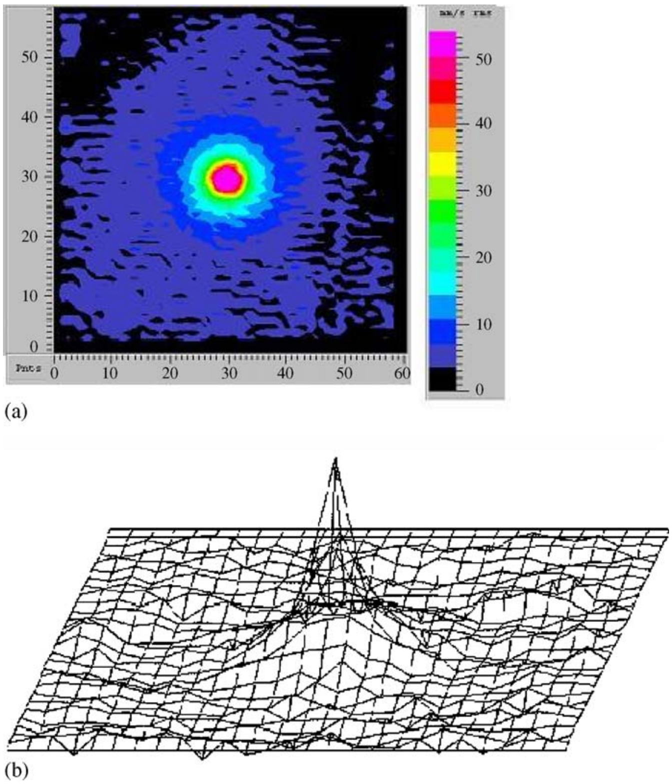


Fig. 4. Membrane vibration shape at frequency = 1430 Hz: (a) velocity contour and (b) corresponding vibration shape of the central region in Fig. 3; the outline of the speaker and the local nature of the response to this excitation are clearly evident.

represent spatial "influence functions" of the local excitation. At 158 Hz, the influence function is broad, spanning most of the membrane, while at 1430 Hz, the function is narrow, not much wider than the excitation pressure zone itself. (Although other vibration modes, and hence power spectrum peaks, may be available for analysis, the two peaks examined above were strong modes and sufficient to illuminate the global/local behavior of membrane vibrations.)

5. Perspective on experimental results

5.1. Membrane vibrations

It may seem worth considering whether the mode localization observed above is a consequence of geometric or material nonlinearities. However, it is shown here that the mode localization tendency can be explained more simply as inherent to the small-amplitude, linear elastic behavior of a membrane. Thus, results are obtained in this section based on linear membrane theory that suggest increasing localization of membrane vibration as a parameter combining excitation frequency, tension, and distance from excitation source is increased. For further simplicity, the effect of air on the vibrations is also ignored, and it is assumed that the membrane vibrates in vacuum and a periodic excitation is provided over a central region by a laser source, for instance. The vibration of a membrane can be considered as the two-dimensional generalization of the vibration of a string, or as the degenerate case of plate vibrations [33,34]. A membrane by definition has vanishing flexural stiffness. Consequently, the "spread" of bending information is weak, but is dependent on the tension, frequency, local curvature, and damping. Here we consider the effect of tension and frequency. Derivation of the governing equations for the linear (small strain, small rotation) transverse vibration of a membrane is amenable to either Newtonian or Lagrangian methods. Using either approach, the initial-boundary value problem of a membrane bounded by a curve $S = S_1 + S_2$ and defined by a unit surface normal **n** becomes

$$T\nabla^2 w + p = \rho \frac{\partial^2 w}{\partial t^2}, \tag{1}$$

with initial conditions w_0 and $\partial w_0/\partial t$, and boundary conditions either fixed along S_1, i.e., $w = 0$, on S_1 and/or free along S_2, i.e., $T(\partial w/\partial n) = 0$, on S_2. Here T is the uniform tension per unit length in the membrane, p is a uniform pressure, ρ is the mass per unit area, w is the transverse displacement, and ∇^2 is the Laplacian operator. (Other boundary conditions are also possible, for example boundary applied forces, but these are not considered here.) As noted, the membrane is assumed theoretically to be operating in vacuum, so that the added mass, stiffness and damping effects of the medium are not considered here. Material damping is also ignored, to focus on the "natural" behavior associated with the inherent stiffness and inertia of the membrane.

The usual method of analysis is to assume a solution separable in space and time, e.g., $w(r,\theta,t) = W(r,\theta)\Psi(t)$. The assumed solution is substituted back into the equation of motion, resulting in two ordinary differential equations, one spatial and the other temporal. Thus the initial-boundary value problem is split into an initial value problem and a boundary value problem. The spatial problem leads to the mode shapes, while the temporal problem determines vibration frequencies.

In the literature, the three classical configurations studied are the circular, rectangular, and triangular shapes. Here we focus on circular membranes, for which the spatial solution is given in terms of Bessel functions J_n. For a circular membrane of radius R fixed at the boundary, the symmetric modes are given by

$$W(r,\theta) = \frac{1}{\sqrt{\pi\rho}RJ_1(\beta_{0m}R)} J_0(\beta_{0m}r), \quad \omega_{0m} = \beta_{0m}\sqrt{\left(\frac{T}{\rho}\right)}, \tag{2}$$

where $m = 1, 2, 3, \ldots, \infty$ and $\beta_{0m}R$ are the zeros of the Bessel function J_0. ω_{0m} is the natural frequency (circular) of the $0m$ mode. Anti-symmetric modes also exist.

The theoretical dynamic displacement of the circular membrane subjected to a harmonic pressure load over central circular area of radius a can be found using [35]

$$w(r,t) = \frac{2a}{R^2}\sqrt{\frac{1}{T\rho}}\sum_{m=1}^{\infty} \frac{J_1(\beta_{0m}a)J_0(\beta_{0m}r)}{\beta_{0m}^2 J_1^2(\beta_{0m}R)} \int_0^t p(\tau)\sin\omega_{0m}(t-\tau)\,\mathrm{d}\tau, \tag{3}$$

where $p(t)$ represents a general time-dependent forcing function. An additional exponential term, $\exp[-\zeta\omega_{0m}(t-\tau)]$ may be added to the above expression to account for damping in the membrane, where ζ represents the structural damping ratio.

We consider below a circular membrane of radius R under tension T (per unit length) and excited by an oscillatory pressure distribution applied over a central area of radius a. The pressure distribution is represented as

$$\begin{aligned} p(r,t) &= p_0 \mathrm{e}^{\mathrm{i}\omega t}, \quad 0 \leqslant r \leqslant a, \\ &= 0, \quad r > a, \end{aligned} \tag{4}$$

where ω denotes the excitation circular frequency. Since the load distribution is axisymmetric, it forces an axisymmetric response in the membrane. Two situations arise depending on the extent of the membrane relative to the area over which the load is applied.

5.2. *Infinite circular membrane*

If the membrane radius R is several orders of magnitude greater than the radius a over which the oscillatory pressure distribution is applied (i.e., $R/a \gg 1$), the membrane may be considered "infinite" in extent. The solution for this case is discussed by Graff [36]. Without loss of generality, we may write the response to a loading corresponding to the real part of $p(r,t)$ as

$$w(r,t) = \frac{p_0 a}{2kT}\frac{J_0(kr)}{J_0(ka)}\cos\omega t. \tag{5}$$

Here $k = 2\pi/\lambda$ is the wavenumber and λ the wave length. The wave propagation speed c_0 is

$$c_0 = \sqrt{\frac{T}{\rho}} = \frac{\omega}{k}. \tag{6}$$

Since p_0, a, and $J_0(ka)$ are all finite and independent of r,

$$w(r,t) \sim W(r)\cos\omega t, \tag{7}$$

where

$$W(r) = \frac{1}{2\omega\sqrt{\rho T}} J_0\left(\omega r\sqrt{\frac{\rho}{T}}\right). \tag{8}$$

In Eq. (7), the term $\cos\omega t$ represents a steady oscillation and is clearly independent of r. $W(r)$ is examined further below.

As $kr = \omega r\sqrt{\rho/T} \to \infty$ (i.e., for large ω, r, and/or small T [37]),

$$J_0\left(\omega r\sqrt{\frac{\rho}{T}}\right) \to \frac{2}{\sqrt{\pi}}\left(\omega r\sqrt{\frac{\rho}{T}}\right)^{-1/2}\cos\left(\omega r\sqrt{\frac{\rho}{T}} - \frac{\pi}{4}\right), \tag{9}$$

so that

$$W(r) \to \frac{1}{(\rho^3\pi)^{1/2}(\omega^6 r^2 T)^{1/4}}\cos\left(\omega r\sqrt{\frac{\rho}{T}} - \frac{\pi}{4}\right). \tag{10}$$

With the cosine term representing steady oscillation in r,

$$W(r) \to 0 \quad \text{if } \omega^6 r^2 T \to \infty \text{ or } \omega r^{1/3} T^{1/6} \to \infty. \tag{11}$$

Therefore, for an infinite membrane, $w(r,t) \to 0$ as long as $\omega r^{1/3}T^{1/6} \to \infty$. Thus, as ω increases for a given tension T, the displacement approaches zero at a point r far enough away from the center. Fig. 5 plots the $W(r)$ distribution as given by Eq. (8), normalized with respect to the maximum value of $W(r)$ over r. To help in the process of visualization of the membrane response, oscillations at a distance $r = 5a$ (corresponding to points far enough away from the excitation region $r \leqslant a$) are observed here. As the parameter $\omega r^{1/3}T^{1/6}$ increases with increasing excitation frequency ω, the amplitude is seen to drop dramatically in Fig. 5. Similar trends are noted at smaller (larger) distances, albeit at larger (smaller) values of the parameter $\omega r^{1/3}T^{1/6}$. The effect of increasing $\omega r^{1/3}T^{1/6}(\to \infty)$ as inferred from Eq. (11) for $R/a \gg 1$ is confirmed by this calculation.

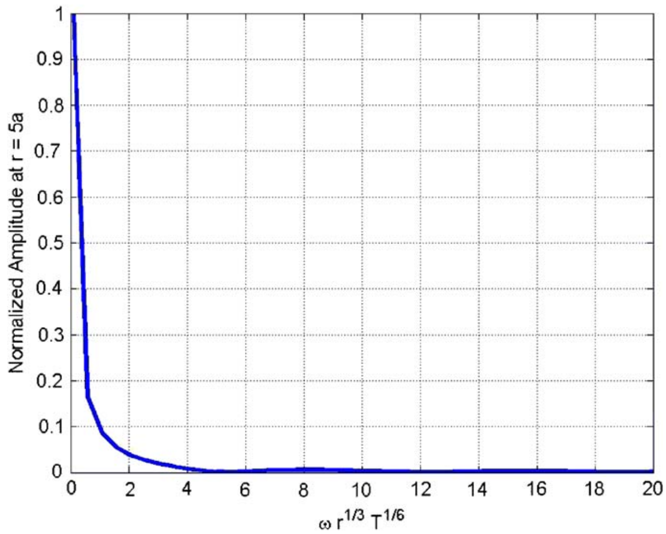


Fig. 5. Amplitude distribution for an infinite membrane as a function of $\omega r^{1/3}T^{1/6}$.

To sum up, at large values of the parameter $\omega r^{1/3}T^{1/6}$, the oscillations appear to be strongly localized as observed for large frequencies in the experiments. Since the theory here is only valid for small amplitudes, it is safe to conclude that the *tendency* for vibration localization at large frequencies is confirmed by these calculations for $R/a \gg 1$.

5.3. Finite circular membrane

If the membrane radius R is not several orders of magnitude greater than a, the membrane boundary conditions now play a role in the dynamics. Using $\psi_{nm}(r,\theta)$ as an eigenfunction for the given boundary conditions, we expand the applied load as

$$p_0 = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} f_{nm}\psi_{nm}(r,\theta), \tag{12}$$

where

$$f_{nm} = \int_0^R \int_0^{2\pi} r p_0 \psi_{nm}(r,\theta)\,\mathrm{d}r\,\mathrm{d}\theta. \tag{13}$$

For the given load, the integral over R extends only up to a. Further, ψ_{nm} are independent of θ due to axial symmetry of the load and the membrane. In addition, because p_0 is constant, all terms except those corresponding to $\psi_{0m} \equiv \psi_m$ will integrate to zero. Consequently,

$$p_0 = \sum_{m=1}^{\infty} f_m \psi_m(r). \tag{14}$$

Further, for a simply supported boundary,

$$\psi_m(r) = \frac{J_0(k_m r)}{\sqrt{\pi\rho} R J_1(k_m R)} \quad \text{where } k_m = \frac{\omega_m}{c_0}. \tag{15}$$

Thus, $f_m \sim p_0 J_1(k_m a)/J_1(k_m R)$, and $k_m = \beta_{0m}$ is used here for convenience. For sinusoidal loading, the particular solution containing the convolution integral in Eq. (3) will reduce to a form

$$w_f(r,t) = \sum_{m=1}^{\infty} C_m \psi_m(r) \mathrm{e}^{\mathrm{i}\omega t}. \tag{16}$$

C_m can then be determined via substitution in the equation of motion as [38]

$$C_m = \frac{c_0^2 f_m}{\omega_m^2 - \omega^2}, \tag{17}$$

where we use the shortened notation ω_m to denote ω_{0m}. If $\omega \to \infty$ without approaching any natural frequency ω_m, then the membrane will perform steady state vibrations, with a spatial distribution determined by $J_0(k_m r)$; i.e., large near the center and decreasing approximately as $r^{-1/2}$ toward the boundary.

If ω also approaches a natural frequency ω_m, the membrane oscillations will increase until nonlinear and dissipation effects become substantial. However, it is discussed below how the spatial variation becomes more localized for large frequencies.

For an initial displacement distribution $w(r,0) = 0$, it can be shown that the overall displacement (homogeneous + particular) as ω approaches a natural frequency ω_m is

$$\lim_{\omega \to \omega_m} w(r,t) = \frac{c_0^2}{2\omega_m} J_0\left(\frac{\omega_m}{c_0} r\right) f_m t \sin \omega_m t. \tag{18}$$

Now as $\omega_m \to \infty$, or as $c_0 \to 0$,

$$w(r,t) \to \frac{T^{5/4}}{\pi^{1/2}\rho^{5/4}\omega^{3/2}r^{1/2}} \cos\left(\omega_m r\sqrt{\frac{\rho}{T}} - \frac{\pi}{4}\right) f_m t \sin \omega_m t. \tag{19}$$

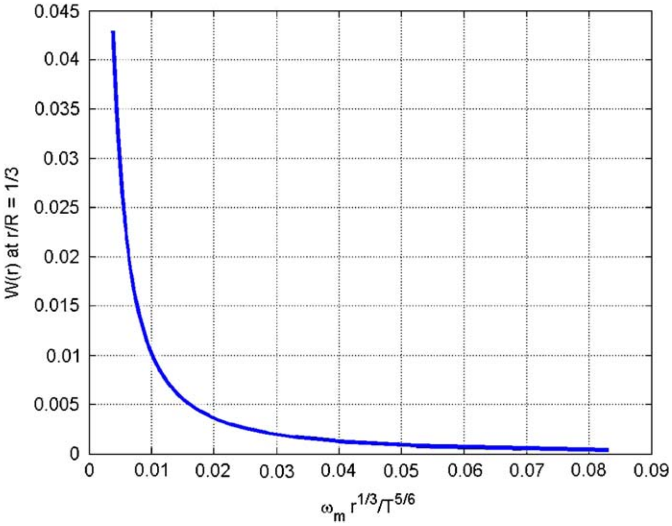


Fig. 6. Amplitude distribution for a finite membrane as a function of $\omega_m r^{1/3}/T^{5/6}$.

Thus,

$$w(r,t) \sim W(r) f_m t \sin \omega_m t \tag{20}$$

and

$$W(r) \to 0 \quad \text{if} \quad \frac{\omega_m r^{1/3}}{T^{5/6}} \to \infty. \tag{21}$$

Although this result ignores material and medium damping, in realistic situations such as studied in our experimental work, the vibration pattern will reach a steady state, and the amplitude distribution will be dominated by $W(r)$. Note that ω_m is here the mth natural frequency of the membrane vibration. Also note that the parameter in Eq. (21) is different from that relevant to infinite membranes, corresponding to the different way in which tension now influences the dynamics of the membrane. A point $r = R/3$ is chosen for the plot in Fig. 6 as a point far enough from the central region of excitation. The figure shows the amplitude variation at a point on a finite membrane at a given time as the ratio $\omega_m r^{1/3}/T^{5/6} \to \infty$. The corresponding decrease in amplitude at a given r confirms that the vibrations are increasingly locally confined at large resonant frequencies for a given tension for finite membranes for which R/a cannot be assumed large. In other words, the membrane resonance becomes more and more localized as the parameter $\omega_m r^{1/3}/T^{5/6}$ increases (i.e., higher frequency or lower tension) as observed experimentally. The analysis of this section and the results above imply that the experimentally observed mode localization tendency can be explained using linear membrane theory. Furthermore, because the theory assumes that the oscillations take place in vacuum, such localization appears to be inherent to the stiffness–inertia related natural dynamics of the membrane. For space-based membrane structures this could have important implications, while also suggesting the possibility of laser-activated local querying at appropriate frequencies for diagnostics.

6. Conclusion

Membranes provide for unique structural response due to their extreme thinness and typically low modulus. Hence the communication of bending information spatially is very weak due to the resultant vanishing flexural stiffness. For a physical analog we can think of a link chain lightly strung between to supports at the same elevation. Information about transverse motion of links in the center of the chain is not communicated well down the length due to the loose coupling between links. The spread of information spatially across the membrane depends on the membrane tension and local curvature, the frequency content of the disturbance, and other factors such as damping.

The extreme structural compliance of membranes makes experiments in membrane vibrations particularly challenging. The small number of membrane vibration experiments reported in the literature is due at least in part to these difficulties. Before the advent of modern technology, such as the laser vibrometer, early investigators developed exceptionally clever methods to make their experimental measurements, and we have reported a concise summary of those here. We then presented details of a recent vibration experiment that discloses a heretofore unreported mode localization response in lightly tensioned membranes.

We have shown that the tendency for mode localization can be explained using linear theory. In particular, we found that when the region of excitation is much smaller than the membrane diameter, oscillations become more and more localized around the region of excitation as the excitation frequency increases for a given tension. In addition, we also showed that when membrane size was finite, the resonant oscillations also tended to become increasingly localized with increase in frequency for a given tension, and that this effect was amplified with reduction in membrane tension. The theoretical results also show that the localization tendency appears to be due to the inherent stiffness–inertia related dynamics of membranes.

The findings reported in this paper could have important implications for space-based membrane structures, and could be applied in localized diagnosis of the stress state and geometry of such structures.

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